percentage of incorrect responses is excluded in an analysis of reaction times [RTs]). We describe three advantages with an experiment on cueing of visual attention.

Firstly and most importantly, we use LMMs to estimate not only effects and interactions of experimental manipulations (i.e., fixed effects parameters), but to estimate simultaneously parameters of the variance and covariance components of random effects due to subjects. Random effects are subjects' deviations from the grand mean RT and subjects' deviations from the fixed-effect parameters. They are assumed to be independently and normally distributed with a mean of 0. It is important to recognize that these random effects are *not* parameters of the LMM – only their variances and covariances are. This LMM feature encapsulates the legacy of Cronbach (1957, 1975).

Secondly, LMMs have much more statistical power than ANOVAs in unbalanced designs. Here we are not referring to lack of balance due to missing data, but due to experimental design. Most notable in this respect are experimental manipulations of cue validity in attention research where trials in which a cue validly indicates the location of a ions of 1(alid)-36xe Tm[/T12 1(alid)-36xe Tm[/T1i0 0 9.3(ynE

belonged to the different-object condition (DOS; "different object, same distance"). Finally, if the target was presented at the other end of the uncued rectangle, this trial was called diagonally different-object condition, or for short, diagonal condition (DOD; "different object, diagonal location"). The four experimental conditions yield three contrasts (in addition to an estimate of the grand mean RT based on the four condition means):

$$\text{Spatial effect} = \text{SOD} - \text{VALID} \quad (1)$$

$$\text{Object effect} = [\text{DOS} - \text{VALID}] - [\text{SOD} - \text{VALID}] = \text{DOS} - \text{SOD} \quad (2)$$

$$\text{Attraction effect} = [\text{DOS} - \text{VALID}] - [\text{DOD} - \text{VALID}] = \text{DOS} - \text{DOD} \quad (3)$$

We specify these three contrasts as *planned comparisons*. In addition, we are interested in the correlation of these effects. As described above, on the admittedly speculative assumption that slow RTs translate into the equivalent of a long SOA, spatial effects should be associated with long mean RTs. Assuming individual differences in the degree to which attention gravitates back to the display centroid, we expect a negative correlation between the spatial and the attraction effects.

A E A CA E E EN A N F

Linear mixed models extend the linear model with the inclusion of random effects, in our case due to differences between subjects. Following Pinheiro and Bates (2000), we use Laird and Ware's (1982) formulation that expresses the n_i-dimensional response vector $\mathbf{y}_i$ for the ith of M subjects as:

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\epsilon}_i,\ i = 1, \quad, \text{M, with } \mathbf{b}_i \quad N(0, \psi),\ \boldsymbol{\epsilon}_i \quad N(0, \sigma^2\boldsymbol{I}) \quad (4)$$

where $\boldsymbol{\beta}$ is the p-dimensional vector of *fixed effect parameters*, $\mathbf{b}_i$ is the q-dimensional vector of *random effects* assumed to be normally distributed with a mean of 0 and a variance–covariance matrix Ψ, and $\boldsymbol{\varepsilon}_i$ is the n_i-dimensional *within-subject error* vector also conforming to a normal distribution. The random effects $\mathbf{b}_i$ and the within-subject errors $\boldsymbol{\varepsilon}_i$ are assumed to be independent for different subjects and to be independent of each other for the same subject.

$\mathbf{X}_i$ with dimensions $n_i \times p$ is the familiar design matrix of the general linear model; $\mathbf{X}_i\boldsymbol{\beta}$ is the overall or fixed component of the model. $\mathbf{Z}_i$ with dimensions $n_i \times q$ is the design matrix for subject i; $\mathbf{Z}_i\mathbf{b}_i$ represents the ran0.5385 65.8542 .3(ne)3 BT/7)-8ff( )]TJETEMC

correlations between the random effects within each subject. It is easy to show that they have an effect on the structure of the covariance matrix $\mathbf{V}_i$ of the response vector $\mathbf{y}_i$. Specifically,

$$\mathrm{y}_i \sim N(\mathrm{X}_i\beta, \mathrm{V}_i), \text{with } \mathrm{V}_i = \mathrm{Z}_i\Psi\mathrm{Z}_i^{\mathrm{T}} + \sigma^2\boldsymbol{I} \quad (5)$$

Thus, random effects will induce a correlation structure between responses of a given subject. A pure random-intercept model with subjects as the random factor yields estimates of the between-subject variance ψ^2 and of the within-subject (residual) variance σ^2; the intraclass coefficient $\psi^2/(\psi^2 + \sigma^2)$ represents the correlation between values of two randomly drawn responses in the same, randomly drawn subject (Snijders and Bosker, 1999).

A CA N U A -A EN NE E EN

For the visual-attention experiment we estimate four fixed-effect parameters ($p = 4$; i.e., intercept and three effects from four experimental conditions). For each of these four parameters we assume reliable differences between the subjects ($q = 4$; $M = 61$; i.e., $4 \times 61 = 244$ random effects). The random effects are parameterized with ten variance/covariance components, that is, with four variances – between-subject variability of mean RT (i.e., random intercept) and between-subject variability of three effects (i.e., random slopes) – and with six correlations of the subject-specific differences in mean RT and three experimental effects. Thus, we estimate a total of 14 model parameters plus the variance of the residual error for the full LMM. Note that the number of parameters grows quadratically with the number of random effects if the full variance–covariance matrix Ψ is estimated. Frequently, one encounters practical limits, primarily related to the amount of information that can be extracted reliably from the data of a psychological experiment. Therefore, the correlation parameters are often forced to 0 or only a theoretically motivated subset of fixed effects $\boldsymbol{\beta}$ is parameterized in the variance–covariance matrix Ψ.

For tests of hypotheses relating to individual differences in experimental effects there are at least three procedures. In the first procedure, groups are defined *post hoc* on median splits on one of the effects or on mean RT; this group factor is included as a between-subject factor in the ANOVA. Then, predictions of correlations map onto group × effect or effect × effect interactions. For example, a positive correlation between mean RT and spatial effect may correspond to a larger *post hoc* group difference on the SOD than the VALID cue condition. The problem with this procedure is that it does not use information about individual differences within each of the *post hoc* groups and typically the dependent variable (i.e., RT) is used to define an independent variable (i.e., it requires a *post hoc* specification of an experimental design factor).

In the second strategy, mean RTs and experimental effects are estimated separately for each subject, for example, with ordinary least-squares regression (i.e., a within-subject analysis of the experimental contrasts). Subsequent correlations between these regression coefficients represent the desired effect correlations. The problem with this procedure is that per-subject regressions accumulate a considerable degree of overfitting (Baayen, 2008). It is also well known, of course, that such within-subject difference scores are notoriously unreliable. With few exceptions, the low reliability of difference scores derived from experimental conditions

conditions, with 48 (10%) per condition. Half of the trials in each condition were with horizontally placed parallel rectangles and half with vertically presented rectangles.

PROCEDURE

Presentation of stimuli and recording of response times and error rates were controlled by Presentation software (http://nbs.neuro-bs.com/).

As a statistical test of the significance of variance/covariance components, we started with a model containing only a directly visible in the figure due to scale differences, prediction intervals of mean RTs are quite a bit narrower than those of the effects due to the fact that the latter are difference scores. Finally, the “implicit slopes” in **Figure 2** for conditional modes across subjects reflect the positive correlation between spatial effect and mean RT.

Second, prediction intervals for the subjects’ object effects overlap very strongly, suggesting that there is not much reliable between-subject variance associated with this effect. Nevertheless, an LMM without variance/covariance components for the object effect fits marginally worse than the complete model, with a $\Delta\chi^2$ (4 df) = 9.5, p = 0.02, for the decrease in loglikelihood. The reported p-value is based on a parametric bootstrap (1000 samples) since the conventional effects. As shown in **Figure 4**, slow subjects showed larger spatial effects [$F(1, 59) = 17.1$, MSE = 264, $p < 0.01$]. The two other interactions were not significant; both F-values < 1. Note that between-subject variance in mean RT was removed for this plot; this, of course, also removes the main effect of speed group. Consequently, it becomes apparent that slow subjects are relatively faster on trials with valid than invalid cue-target relations. Thus, the results are in agreement with the expectation that the spatial effect is modulated by individual differences in RT. Slow subjects engage attention more at the cued location than fast subjects. There is no significant evidence for such a modulation for object and attraction effects. The corresponding LMM correlation parameters were +0.60, −0.13, and −0.25 (see **Table 2**).

Attraction effect as post hoc grouping factor

We also predicted that subjects with an attraction effect will show a comparatively small spatial effect. For a *post hoc* ANOVA test of this hypothesis, we classified subjects according to whether or cueing effect in our study was augmented by the simultaneous activation of an object – an issue that should be addressed in future studies.

Importantly and in support of the assumption that individual differences in RT may represent a quasi-experimental SOA manipulation (Lamy and Egeth, 2002

Cronbach, L. J. (1975). Beyond the two disciplines of scientific psychology. *Am. Psychol.* 30, 116–127.

Egly, R., Driver, J., and Rafal, R. D. (1994). Shifting visual attention between objects and locations: evidence from normal and parietal lesion subjects. *J. Exp. Psychol. Gen.* 123, 161–177.

Egly, R., and Homa, E. (1991). Reallocation of attention. *J. Exp. Psychol. Hum. Percept. Perform.* 17, 142–159.

Eriksen, C. W., and St. James, J. D. (1986). Visual attention within and around